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Aspects of Soliton Propagation in Stimulated Raman Scattering

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Solitons have recently been observed in stimulated Raman scattering (SRS) (details about the experiments and the identification of solitons may be found in Refs. 1 and 2). In this paper we shall discuss certain aspects concerning the propagation of solitonlike excitations through a finite medium in the presence of coherence decay, and for optical pulses of finite duration. The equations for SRS are given by

$$\partial X/\partial \tau = -\varepsilon X + A_1 A_2^* \tag{1}$$

$$\partial A_1/\partial \zeta = -X A_2 \tag{2}$$

$$\partial A_2/\partial \zeta = X^* A_1 \tag{3}$$

X is the off-diagonal matrix element for the Raman transition, corresponding to the coherent polarization induced by the optical fields. The first term in Eq. (1) describes collisional decay of phase coherence (damping) with rate ε. A corresponding equation for the diagonal matrix element, giving the population difference, has been suppressed, since this difference remains nearly constant under the given experimental conditions. A_1 and A_2 are the slowly varying envelopes of the pump and Stokes electric fields. τ and ζ are related to time t and position z in the laboratory by

$$\zeta = z, \qquad \tau = t - z/c \tag{4}$$

Suitable units of intensity, length, and time have been chosen to render all coupling constants equal to unity. Note that the unit of time in (1) has been chosen to represent an intrinsic, typical time scale of variation for the

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class of solutions considered ($\partial X/\partial \tau \approx X$). The dimensionless parameter ε hence may be considered as this intrinsic time scale measured in units of the damping time (inverse damping rate).

For a finite medium extending into the half-plane $Z>0$ and optical fields of finite duration, which vanish inside the medium at $t=0$, the initial conditions are

$$A_k(Z=0, t)=A_k(\zeta=0, \tau=t)=a_k(\tau) \qquad k=1,2 \tag{5a}$$

$$a_k(\tau)=0 \qquad \text{for} \quad \tau<0 \qquad k=1,2 \tag{5b}$$

$$X(Z, t=0)=0 \qquad \text{for} \quad Z>0 \tag{6a}$$

$$X(Z, t=c/Z)=X(\zeta=Z, \tau=0)=0 \tag{6b}$$

(6b) is a consequence of Eqs. (1)–(3) and Eq. (5b), and expresses the fact that no polarization is induced in the medium before the arrival of the leading edge of the optical pulses.

Solutions to Eqs. (1)–(3) for $\varepsilon=0$ have been found by Chu and Scott,[3,4] and recently for a more complete set of equations by Kaup[5] and Steudel.[6] Chu and Scott give in particular the one-soliton form of the matrix element X (which equals their Y except for a phase factor). From their solution and Eqs. (1)–(3) the optical fields can be calculated to give the complete solution:

$$X=\mu_I e^{i\alpha(\zeta,\tau)} \operatorname{sech} \beta(\zeta, \tau) \tag{7a}$$

$$A_1=\mu_I e^{i\alpha(\zeta,\tau)} \operatorname{sech} \beta(\zeta, \tau)/(\mu_I^2+\mu_R^2)^{1/2} \tag{7b}$$

$$A_2=[\mu_I \tanh \beta(\zeta, \tau)-i\mu_R]/(\mu_I^2+\mu_R^2)^{1/2} \tag{7c}$$

where $\alpha(\zeta, \tau)=\mu_R\zeta+\omega_I\tau$, $\beta(\zeta, \tau)=\mu_I\zeta-\omega_R\tau+\beta_0$, $\omega_R=\mu_I/(\mu_I^2+\mu_R^2)$ and $\omega_I=\mu_R/(\mu_I^2+\mu_R^2)$.

μ_I and μ_R are constants of integration. The solution is essentially a traveling wave with group velocity v in the laboratory, where $v^{-1}=c^{-1}+\mu_I^2+\mu_R^2$. Its temporal width is given by $\Delta\tau=\omega_R^{-1}$. Stokes and pump frequency difference are detuned from Raman resonance by $\Delta\omega=\omega_I$.

The sum of $|A_1|^2$ and $|A_2|^2$ is equal to unity. At the center of the excitation ($\beta=0$) the pump intensity assumes its maximum value

$$|A_1|^2_{\max}=\mu_I^2/(\mu_R^2+\mu_I^2)=1/[1+(\Delta\omega\Delta\tau)^2] \leqslant 1$$

while the Stokes field undergoes a rapid phase shift of $\Delta\phi=2\tan^{-1}(1/\Delta\omega\Delta\tau)$.

This solution does not satisfy the boundary conditions (5) and (6). In fact it was found by the method of inverse spectral transform (IST) (see, for example, Ref. 7), where the field values are assumed to be given for all ζ at $\tau = 0$, and their values for $\tau > 0$ are then calculated.

Nevertheless a class of solutions (7) satisfies Eqs. (5) and (6) to a very good approximation, and is hence of physical relevance. Firstly we observe that Eq. (7) remains a valid solution when restricted to a finite τ interval. Secondly, for $\beta_0 \gg 1$, $X(\zeta, \tau = 0)$ will be exponentially small for $\zeta > 0$ and Eq. (6a) is satisfied approximately. Note, however, that for any finite τ interval the soliton will disappear at the trailing edge of the optical pulses after a finite propagation distance, owing to its reduced group velocity $v < c$. This may explain why no observation of solitons has been reported so far in hypertransient SRS ($\varepsilon \ll 0$).

The physical mechanism of soliton propagation becomes most evident for the case $\mu_R = 0$ (no detuning). In this case the matrix element X and the driving term $A \cdot A_2^*$ will both have the same sign in the leading edge of the soliton ($\beta > 0$), leading to loss for the pump and gain for the Stokes field by Eqs. (2) and (3). In the trailing edge ($\beta < 0$), however, the driving term has changed sign, and is out of phase by 180° with the matrix element X. Now loss and gain are reversed; the pump field sees gain, and the Stokes field sees loss. In physical terms this is anti-Stokes scattering. Normally anti-Stokes scattering occurs because of population inversion in the Raman active medium, leading to a change of sign in the right-hand side of Eq. (1). In this case the necessary phase relation is created dynamically by the rapid change of phase in the Stokes field, without any population inversion. Note that this effect is quite analogous to the occurrence of stimulated emission in the trailing edge of a bleaching wave in self-induced transparency.[8]

For $\varepsilon > 0$ the same physical mechanism will lead to the occurrence of solitonlike solutions, which are, however, no longer traveling waves. A perturbative analysis within the general framework of the Zakharov–Shabat inverse scattering scheme has been given by Kaup.[9] He assumes that the field variables (in our case the matrix element X) are given by the one-soliton solution for all ζ at $\tau = 0$, and derives equations for the time dependence of the solution parameters valid to first order in ε. From his general analysis we obtain the following solution to Eqs. (1)–(3), for $\mu_R = 0$, valid to first order in ε:

$$X(\zeta, \tau) = \mu_I(\tau) \operatorname{sech} \beta_\varepsilon(\zeta, \tau) + \Delta X(\zeta, \tau) \tag{8a}$$

$$\beta_\varepsilon(\zeta, \tau) = \mu_I(\tau)\, \zeta - \frac{1}{\mu_I(0)} \frac{1}{2\varepsilon} \sinh 2\varepsilon\tau \tag{8b}$$

$$\mu_I(\tau) = \mu_I(0)\, e^{-2\varepsilon\tau} \tag{8c}$$

While this approach again does not solve the physical boundary value problem where the fields are assumed to be given for all τ at $\zeta = 0$, we can nevertheless derive some useful conclusions about the ζ dependence of the observed excitation from the solution above. The temporal position of the soliton center is given by

$$\tau = \tau_0(\zeta), \qquad \beta(\zeta, \tau_0(\zeta)) = 0$$
$$\exp[4\varepsilon\tau_0(\zeta)] = 4\varepsilon\mu_I^2(0)\zeta + 1 \tag{9}$$

Expanding β_ε to first order in ζ and τ at $\tau = \tau_0$ we obtain the one-soliton form for the first term in (8a) with τ-dependent parameter:

$$X(\zeta + \zeta', \tau_0 + \tau') \simeq \mu_I(\tau_0) \operatorname{sech} \beta(\tau_0; \zeta', \tau') + \Delta X(\zeta + \zeta', \tau_0 + \tau')$$
$$\beta(\tau_0; \zeta', \tau') = \mu_I(\tau_0)\, \zeta' - \mu_I^{-1}(\tau_0)\, \tau' \tag{10}$$

This expansion is valid for sufficiently small parameter $\mu_I(0)$. These results show that the temporal width of the soliton will narrow down with increasing gain length ζ approximately like the inverse square root of ζ. Furthermore, the center position of the soliton in the enveloping optical pulse will depend only weakly (logarithmically) on the gain length, showing that the soliton velocity approaches the velocity of light in the laboratory frame. Both conclusions are confirmed by experimental[2] and numerical studies.

In Fig. 1 we show numerical results of the temporal soliton position and width, together with theoretical results from (9) and (10). The soliton parameter is equal to $\mu_I(0) = 0.112$ and was obtained from the slope of curve 1. The agreement is good, although not perfect. A reformulation of the inverse spectral transformation scheme for this case, which allows treatment of the physical boundary value problem, is presently under study.

In the present numerical experiments and also in the laboratory experiments[2] the solitons are created by a sudden phase change of 180° in the injected Stokes beam. This occurs at $\zeta = 0$, that is, before any amplification in the medium has taken place, and the Stokes intensity is typically several orders of magnitude smaller than the pump intensity. In the presence of coherence decay and for exact resonance this configuration leads to the buildup of a one-soliton excitation as the pulse is propagated into the medium ($\zeta > 0$), which shows the features of temporal narrowing and acceleration in the laboratory frame, as discussed above. In the absence of coherence decay, and for nonzero detuning ($\mu_R \neq 0$) the situation is less clear at the moment. Our numerical experiments indicate that for $\varepsilon = 0$ no solutions with well-defined one-soliton features will develop at zero detuning for values of ζ which are accessible experimentally. For $\mu_R \neq 0$ both experimental[2] and numerical[10] results indicate that solitonlike solutions do develop initially; however, they will decay at

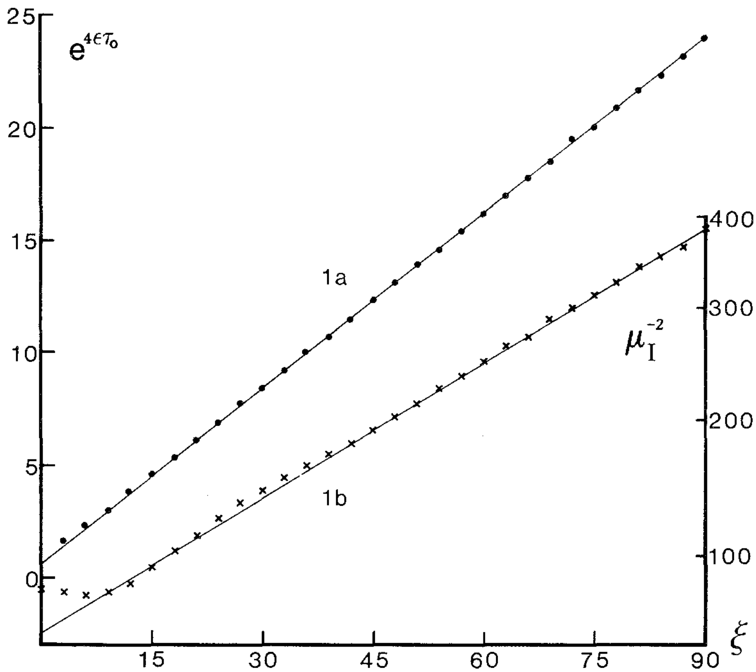

Fig. 1. Curve 1: Soliton position $\exp 4\varepsilon\tau_0$; solid line, analytical approximation; ○, numerical. Curve 2: Soliton width $\Delta\tau(\zeta)/\Delta\tau(0)$; solid line, analytical approximation; ×, numerical.

larger ζ for a large class of initial conditions. This result seems plausible in view of the fact that the solution (7a) to (7c) imposes certain relations between detuning $\Delta\omega$, soliton width $\Delta\tau$, total phase shift $\Delta\phi$, and maximal pump amplitude. Both questions are presently under study within an inverse spectral transform approach to the physical initial conditions (5) and (6).[11]

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